

Comment on “Network analysis of the state space of discrete dynamical systems”

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In paper [1], the authors analyzed the state space network of discrete dynamical systems composed by 1-D cellular automata (CA) with rules involving two colors and nearest neighbors, and claimed that “the scaling and distribution of in-degrees and the path diversity give a good indication of dynamical complexity”. Concretely, the statement means that the co-appearance of nontrivial scaling in both the hub size and the path diversity of the phase networks can separate simple dynamics from the more complex ones found in CA falling in Wolfram’s class III, IV. This Comment is to point out that analytic results for rule 4 are wrong and the proposed indicators can not discriminate dynamical complexity of cellular automata at all.

In [1], there are some obvious defects on estimation of in-degree distribution of phase network corresponding to rule 4. The authors adopted $a\lambda^{m_i}$ as approximate value of $w(m_i)$. Unfortunately, the estimation error is accumulated exponentially in the multiplication calculation of $k_n = \prod_{i=1}^n w(m_i) = \prod_{i=1}^n (a\lambda^{m_i})$. The authors of [1] calculate $\Omega(n)$, the number of states with n isolated 1’s and no pairs ‘11’, as $C(L-n, n) + C(L-n-1, n-1)$. Actually, $\Omega(n) = C(L-n+1, n) = C(L-n, n) + C(L-n, n-1)$. As for rule 4, every non-isolated node satisfies $L-n+1 \geq n$. So, Eq. (7) in [1] outputs complex number as $(1-2\epsilon) \leq 0$ when $L \leq 2n \leq L+1$, where $\epsilon \equiv n/L$. The cascaded effect of these defects make Eq. (7) can not accurately approximate distribution of the largest in-degree. In addition, the scope of Y-axis of Fig. 4 in [1] should be $[-1, 0]$ not $[-2, 0]$, which can be deduced by $1/N \leq P(k) \leq 1$, and $-1 \leq \log P(k)/\log N \leq 0$. We re-calculated the in-degree distribution functions for three rules under three systems sizes in Fig. 1 and re-draw Eq. (7) at its left part as a bold black curve, which support our findings.

As for the 1-D CA under study, the 256 elementary rules are reduced to 88 independent rules under some transformation [2, Page 294]. However, only ten independent rules were selected to verify the statement in [1]. We calculated the change slope of largest in-degree and path diversity with respect to CA size for every independent rule and depict the results in Fig. 2, which is divided into four panels according to Wolfram’s classification (The concrete classification specification is referred in [3, Page 231], [2, Table 1], and [4, Table 3]). From each panel, no general law can be observed. Furthermore, we can see that both the two observed values are very close among rule 54, 62, 110, and 182, which belonging to three different classes. More counterexamples

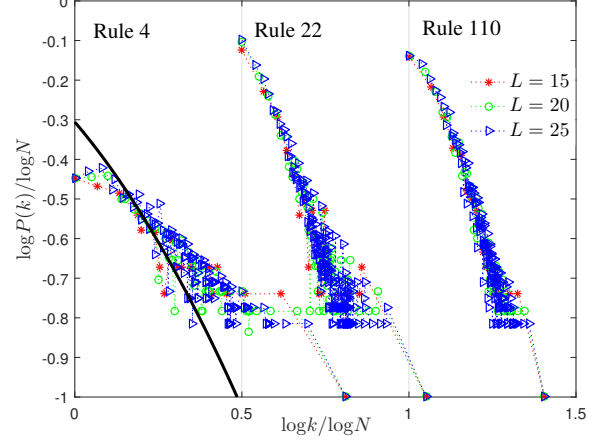


FIG. 1. In-degree distribution functions for three rules under different system sizes.

can be found to verify inefficiency of the main statement of [1] from Fig. 2.

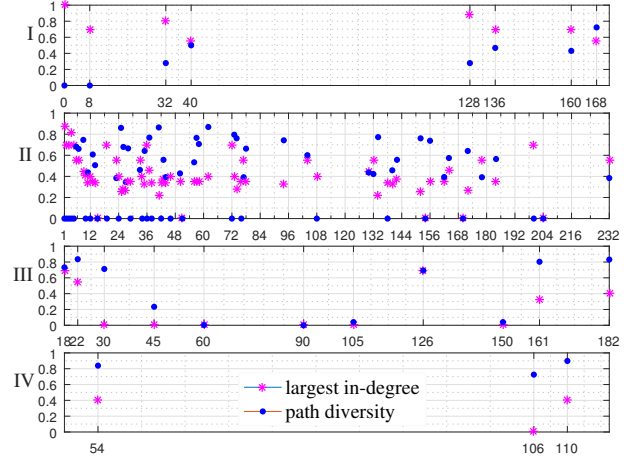


FIG. 2. Distribution of the two observables defined in [1] with respect to rule number.

In conclusion, the in-degree distribution and the path diversity of phase network can not measure dynamical complexity of the systems formed by 1-D CA. Justification for measuring complexities of general discrete dynamical systems using statistical characteristics of their phase networks needs further investigation [5–7].

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